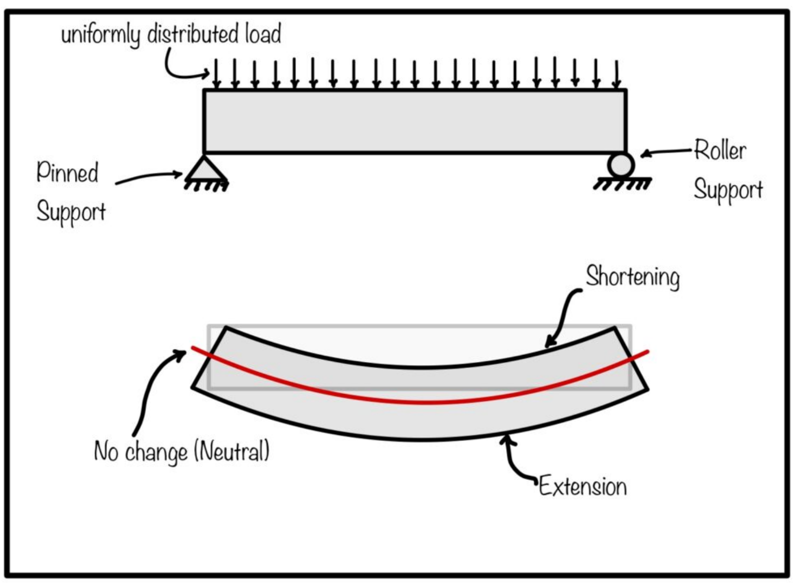
Bending stress in beams

Introduction:

Beams in a structure are generally subjected to different types external loads. Due to these external loads shear force and bending moments are developed in a beam which results in deformation. Beam material develops certain types of resistance forces or stresses against this deformation. **These stresses in the beam due to the bending moment are called as bending stresses.**

Definition:

When a beam is subjected to **simple or pure bending**, (I.e. shear force is zero) the stresses developed are due to bending moment alone. The stresses formed due to bending moment are called as bending stresses. However certain assumptions are made in theory of simple bending.



Bending stress is the internal stress induced in a beam when an external bending moment is applied, causing the beam to deform. It varies linearly across the cross-section, being maximum at the outermost fibres and zero at the neutral axis.

Formula:

$\sigma = (M \times y) / I$

Where:

- σ = Bending stress (N/m^2 or Pa)
- M = Bending moment at the section (Nm)
- y = Distance from the neutral axis to the point where the stress is calculated (m)
- I = Moment of inertia of the cross-section about the neutral axis (m^4)

Assumptions in Bending Theory:

1. The material is homogeneous and isotropic.
2. The beam is initially straight.
3. The cross-section remains plane before and after bending.
4. The radius of curvature is large compared to the dimensions of the cross-section.

Derivation of Bending Stress Formula:

Consider a beam subjected to a bending moment M. Due to bending, the top fibers compress and the bottom fibers extend. There exists a layer in the middle called the neutral axis where the stress is zero.

The strain at a distance y from the neutral axis is given by: Strain = y / R, where R = radius of curvature.

By Hooke's law, Stress = E × Strain: $\sigma = E \times (y / R)$

The bending moment is related to stress by: $M = \int A \, \sigma \, y \, dA$

Substituting $\sigma = E \times (y / R)$: $M = (E / R) \times \int A \, y^2 \, dA = (E / R) \times I$

Rearranging: $M / I = E / R$

From hooks law : $\sigma / y = E / R$

Equating the two: $M / I = \sigma / y$

Therefore: $\sigma = (M \times y) / I$

Applications

- Used in designing beams, bridges, and structural components.
- Helps determine safe load limits to prevent failure.

Bending Stress in Unsymmetrical Sections

In **unsymmetrical beam sections** (like L-sections, T-sections, angle sections), the **neutral axis (N.A.)** doesn't lie at the geometric center, and calculating **bending stress** involves more care.

For **unsymmetrical sections**, such as an L or T section:

1. **Centroid is off-center** → Must find the exact location of the neutral axis (N.A.)
2. **Moment of inertia I**must be calculated about the N.A., not about the geometric center.
3. The **maximum bending stress** occurs at the point **farthest from the N.A.**, not necessarily at the geometric top or bottom.

Steps to Calculate Bending Stress for Unsymmetrical Sections:

1. **Divide the section into simple shapes** (rectangles, etc.)
2. **Calculate area and centroid ($\bar{Y}$)** for each shape.
3. **Find the combined centroid** of the full section using:

$$\bar{Y} = \sum A_i / \sum A_i y_i$$

4. **Use the parallel axis theorem** to calculate the **moment of inertia Ix** about the centroidal axis.
5. Find **maximum distance y** from the N.A. to the outermost fiber.
6. Apply the bending stress formula

examples

A timber cantilever 200 mm wide and 300 mm deep is 3 m long. It is loaded with a U.D.L. of 3 kN/m over the entire length. A point load of 2.7 kN is placed at the free end of the cantilever. Find the maximum bending stress produced.

Given:

Width, b = 200 mm

Depth, d = 300 mm

Length, L = 3 m = 3000 mm

UDL, w = 3 kN/m = 3 N/mm

Point load, P = 2.7 kN = 2700 N

Solution:

Step 1: Find the maximum bending moment

Moment due to point load at free end:

$M_P = P \times L = 2700 \times 3000 = 8.1 \times 10^6$ Nmm

Moment due to UDL over entire span:

$M_{UDL} = (w \times L^2) / 2 = (3 \times 3000^2) / 2 = 13.5 \times 10^6$ Nmm

Total Moment:

$M_{total} = M_P + M_{UDL} = 8.1 \times 10^6 + 13.5 \times 10^6 = 21.6 \times 10^6$ Nmm

Step 2: Calculate Section Modulus (Z)

$Z = (b \times d^2) / 6 = (200 \times 300^2) / 6 = 3 \times 10^6\ mm^3$

Step 3: Calculate Bending Stress

$\sigma = M / Z = 21.6 \times 10^6 / 3 \times 10^6 = 7.2\ N/mm^2$

Answer:

Maximum Bending Stress = $7.2\ N/mm^2$